Investigation of the anomaly puzzle in $N = 1$ supersymmetric electrodynamics.

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Abstract

Using Schwinger-Dyson equations and Ward identities in $N = 1$ supersymmetric electrodynamics, regularized by higher derivatives, we find, that it is possible to calculate some contributions to the two-point Green function of the gauge field and to the β-function exactly to all orders of the perturbation theory. The results are applied for the investigation of the anomaly puzzle in the considered theory.

1 Introduction.

The investigation of quantum corrections in supersymmetric theories is a very interesting and sometimes nontrivial problem. For example, we remind of the so called "anomaly puzzle". The matter is that in supersymmetric theories the axial anomaly and the anomaly of the energy-momentum tensor trace are components of a single chiral supermultiplet [1, 2, 3, 4]. According to Adler-Bardeen theorem [5, 6] the axial anomaly is exhausted at the one-loop, while the trace anomaly should be proportional to the β-function [7] to all orders of the perturbation theory. Therefore the β-function in supersymmetric theories can be supposed to be completely defined by the one-loop approximation [8]. However explicit perturbative calculations in $N = 1$ supersymmetric theories regularized by the dimensional reduction [9] revealed, that there were higher loops contributions to the β-function [10, 11, 12]. Thus we obtain a contradiction, which was called in the literature "the anomaly puzzle".

Different at the first sight solutions of the anomaly puzzle were proposed in [13] and [14]. Nevertheless the results of both papers were not confirmed by explicit calculations so far the dimensional reduction was used for the regularization. The reason, found in

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[15], is the mathematical inconsistency of the dimensional reduction, which was first pointed in [16]. (Note, that unlike the dimensional reduction, the dimensional regularization [17] is not mathematically inconsistent. However it breaks the supersymmetry and is, therefore, very inconvenient for using in supersymmetric theories.) The calculations, made with the mathematically consistent higher derivative regularization [18, 19] in the two- [20, 21] and three-loop [22] approximations for the $N = 1$ supersymmetric electrodynamics, showed, that in this case there was no anomaly puzzle. According to this papers in supersymmetric theories the β-function, defined as a derivative of the renormalized coupling constant with respect to $\ln \mu$, and Gell-Mann-Low function are different due to the rescaling anomaly. The former is proportional to the trace anomaly, while the latter has corrections in all orders of the perturbation theory and in the considered approximation coincides with the exact Novikov, Shifman, Vainshtein and Zakharov (NSVZ) β-function. For the $N = 1$ supersymmetric electrodynamics the exact NSVZ β-function is:

$$\beta(\alpha) = \frac{\alpha^2}{\pi}\Big(1 - \gamma(\alpha)\Big), \tag{1}$$

where $\gamma(\alpha)$ is the anomalous dimension of the matter superfield. (First time the exact β-function was constructed in [23] as a result of the investigation of instanton contributions structure.)

Note, that the using of the higher derivative regularization allows to relate various formal solutions of the anomaly puzzle, which were proposed in the literature.

Nevertheless, the investigation of the supersymmetric electrodynamics up to now was restricted by the frames of the three-loop approximation. However, it would be interesting to elucidate if it is possible to perform the explicit calculations exactly to all orders of the perturbation theory. In this paper we investigate a question, if it is possible to obtain the result, which is exact to all orders, using Schwinger-Dyson equations and Ward identities.

The paper is organized as follows:

In Sec. 2 some information about the $N = 1$ supersymmetric electrodynamics and its regularization by higher derivatives is recalled. Schwinger-Dyson equations for the considered theory are constructed in Sec. 3, Ward identities and their solutions are presented in Sec. 4. The solutions of Ward identities are substituted into the Schwinger-Dyson equation for the two-point Green function of the gauge field in Sec. 5. Then it turned out, that this Green function in the limit $p \to 0$ can be thus calculated almost completely. Making a special proposal about the structure of the remaining contributions it is possible to construct this function completely (Sec. 6). Then divergences in the two-point Green function are present only in the one-loop approximation, but Gell-Mann-Low function coincides with the exact NSVZ β-function. In the conclusion we discuss the obtained results.

2 $N = 1$ supersymmetric electrodynamics and its regularization by higher derivatives

$N = 1$ supersymmetric electrodynamics in the superspace is described by the following action:

$$S_0 = \frac{1}{4e^2}\mathrm{Re}\int d^4x\,d^2\theta\,W_a C^{ab} W_b + \frac{1}{4}\int d^4x\,d^4\theta\left(\phi^* e^{2V}\phi + \tilde{\phi}^* e^{-2V}\tilde{\phi}\right) +$$
$$+\frac{1}{2}\int d^4x\,d^2\theta\,m\,\tilde{\phi}\,\phi + \frac{1}{2}\int d^4x\,d^2\bar{\theta}\,m\tilde{\phi}^*\phi^*. \tag{2}$$

Here ϕ and $\tilde{\phi}$ are chiral matter superfields with the mass m, and V is a real scalar superfield, which contains the gauge field A_μ as a component. The superfield W_a is a supersymmetric analog of the stress tensor of the gauge field. In the Abelian case it is defined by

$$W_a = \frac{1}{16}\bar{D}(1-\gamma_5)D\left[(1+\gamma_5)D_a V\right], \tag{3}$$

where

$$D = \frac{\partial}{\partial\bar{\theta}} - i\gamma^\mu\theta\,\partial_\mu \tag{4}$$

is a supersymmetric covariant derivative.

In order to regularize model (2) it is possible to add to its action the term with the higher derivatives:

$$S_0 \to S = S_0 + S_\Lambda = \frac{1}{4e^2}\mathrm{Re}\int d^4x\,d^2\theta\,W_a C^{ab}\left(1 + \frac{\partial^{2n}}{\Lambda^{2n}}\right)W_b + \tag{5}$$
$$+\frac{1}{4}\int d^4x\,d^4\theta\left(\phi^* e^{2V}\phi + \tilde{\phi}^* e^{-2V}\tilde{\phi}\right) + \frac{1}{2}\int d^4x\,d^2\theta\,m\,\tilde{\phi}\,\phi + \frac{1}{2}\int d^4x\,d^2\bar{\theta}\,m\,\tilde{\phi}^*\phi^*.$$

It is important to note, that in the Abelian case the superfield W^a is gauge invariant, so that there are the ordinary derivatives instead of the covariant ones in the regularizing term.

The quantization of model (5) can be made using the standard methods. For this purpose it is convenient to use the supergraphs technique, described in [24] in details, and to fix the gauge invariance by adding the following terms:

$$S_{gf} = -\frac{1}{64e^2}\int d^4x\,d^4\theta\left(V D^2\bar{D}^2\left(1 + \frac{\partial^{2n}}{\Lambda^{2n}}\right)V + V\bar{D}^2 D^2\left(1 + \frac{\partial^{2n}}{\Lambda^{2n}}\right)V\right), \tag{6}$$

where

$$D^2 \equiv \frac{1}{2}\bar{D}(1+\gamma_5)D; \qquad \bar{D}^2 \equiv \frac{1}{2}\bar{D}(1-\gamma_5)D. \tag{7}$$

After adding such terms a part of the action, quadratic in the superfield V will have the simplest form

$$S_{gauge} + S_{gf} = \frac{1}{4e^2}\int d^4x\,d^4\theta\,V\partial^2\left(1 + \frac{\partial^{2n}}{\Lambda^{2n}}\right)V. \tag{8}$$

In the Abelian case, considered here, diagrams, containing ghost loops are absent.

It is well known (see e.f. [20]), that adding of the higher derivative term does not remove divergences from one-loop diagrams. In order to regularize them, it is necessary to insert in the generating functional the Pauli-Villars determinants [6].

Due to the supersymmetric gauge invariance

$$V \to V - \frac{1}{2}(A + A^+); \qquad \phi \to e^A \phi; \qquad \tilde{\phi} \to e^{-A} \tilde{\phi}, \qquad (9)$$

where A is an arbitrary chiral scalar superfield, the renormalized action can be written as

$$S_{ren} = \frac{1}{4e^2} Z_3(e, \Lambda/\mu) \, \mathrm{Re} \int d^4x \, d^2\theta \, W_a C^{ab} \Big(1 + \frac{\partial^{2n}}{\Lambda^{2n}}\Big) W_b +$$

$$+ Z(e, \Lambda/\mu) \frac{1}{4} \int d^4x \, d^4\theta \, \Big(\phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi}\Big) +$$

$$+ \frac{1}{2} \int d^4x \, d^2\theta \, m \, \tilde{\phi} \phi + \frac{1}{2} \int d^4x \, d^2\bar{\theta} \, m \tilde{\phi}^* \phi^*, \qquad (10)$$

where we take into account, that due to the nonrenormalization theorem [24] the mass term is not renormalized in the perturbation theory. Therefore, the generating functional can be written as

$$Z = \int DV \, D\phi \, D\tilde{\phi} \prod_i \Big(\det PV(V, M_i) \Big)^{c_i} \exp \Big(i(S_{ren} + S_{gf} + S_S + S_{\phi_0}) \Big), \qquad (11)$$

where the renormalized action S_{ren} is given by Eq. (10), the gauge fixing action – by Eq. (6) (It is convenient to substitute e by e_0 in Eq. (6), that we will assume below), the Pauli-Villars determinants are defined by

$$\Big(\det PV(V, M) \Big)^{-1} = \int D\Phi \, D\tilde{\Phi} \, \exp \Big(iS_{PV} \Big), \qquad (12)$$

where

$$S_{PV} \equiv Z(e, \Lambda/\mu) \frac{1}{4} \int d^4x \, d^4\theta \, \Big(\Phi^* e^{2V} \Phi +$$

$$+ \tilde{\Phi}^* e^{-2V} \tilde{\Phi} \Big) + \frac{1}{2} \int d^4x \, d^2\theta \, M \tilde{\Phi}\Phi + \frac{1}{2} \int d^4x \, d^2\bar{\theta} \, M \tilde{\Phi}^* \Phi^*, \qquad (13)$$

and the coefficients c_i satisfy conditions

$$\sum_i c_i = 1; \qquad \sum_i c_i M_i^2 = 0. \qquad (14)$$

Below we will assume, that $M_i = a_i \Lambda$, where a_i are some constants. Insertion of the Pauli-Villars determinants allows to cancel the remaining divergences in all one-loop diagrams, including diagrams, containing insertions of counterterms.

The source terms are written as

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$$S_S = \int d^4x \, d^4\theta \, JV + \int d^4x \, d^2\theta \left(j \, \phi + \tilde{j} \, \tilde{\phi} \right) + \int d^4x \, d^2\bar{\theta} \left(j^* \phi^* + \tilde{j}^* \tilde{\phi}^* \right). \quad (15)$$

Moreover, we introduce into generating functional (11) the expression

$$S_{\phi_0} = \frac{1}{4} \int d^4x \, d^4\theta \left(\phi_0^* \, e^{2V} \phi + \phi^* \, e^{2V} \phi_0 + \tilde{\phi}_0^* \, e^{-2V} \tilde{\phi} + \tilde{\phi}^* \, e^{-2V} \tilde{\phi}_0 \right), \quad (16)$$

where ϕ_0, ϕ_0^*, $\tilde{\phi}_0$ and $\tilde{\phi}_0^*$ are scalar superfields. They are some parameters, which are not chiral or antichiral. In principle, it is not necessary to introduce the term S_{ϕ_0} into the generating functional, but the presence of the parameters ϕ_0 e t.c. will be useful for us later for the investigation of Schwinger-Dyson equations.

In our notations the generating functional for the connected Green functions is

$$W = -i \ln Z, \quad (17)$$

and an effective action is obtained by making a Legendre transformation:

$$\Gamma = W - \int d^4x \, d^4\theta \, JV - \int d^4x \, d^2\theta \left(j \, \phi + \tilde{j} \, \tilde{\phi} \right) - \int d^4x \, d^2\bar{\theta} \left(j^* \phi^* + \tilde{j}^* \tilde{\phi}^* \right), \quad (18)$$

where J, j and $\tilde{j}$ is to be eliminated in terms of the fields V, ϕ and $\tilde{\phi}$, through solving equations

$$V = \frac{\delta W}{\delta J}; \qquad \phi = \frac{\delta W}{\delta j}; \qquad \tilde{\phi} = \frac{\delta W}{\delta \tilde{j}}. \quad (19)$$

3   Schwinger-Dyson equations for $N = 1$ supersymmetric electrodynamics

We will try to calculate the two-point Green function of the gauge field. First for the simplicity we set the renormalization constant Z in Eq. (10) equal to 1, that corresponds to taking into account all diagrams, which do not contain insertions of counterterms on lines of the matter superfields. Later we will see, that if $Z \neq 1$, the two-point Green function of the gauge field can be easily found from the corresponding result with $Z = 1$.

First we should write Schwinger-Dyson equations for the considered theory ($Z = 1$). For this purpose first we split the action into two parts:

$$S = S_2 + S_I, \quad (20)$$

where

$$S_2 \equiv \frac{1}{4e_0^2} \int d^4x \, d^4\theta \, V \partial^2 \left(1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V + \frac{1}{4} \int d^4x \, d^4\theta \left(\phi^* \phi + \tilde{\phi}^* \tilde{\phi} \right) +$$
$$+ \frac{1}{2} \int d^4x \, d^2\theta \, m \, \tilde{\phi} \, \phi + \frac{1}{2} \int d^4x \, d^2\bar{\theta} \, m \tilde{\phi}^* \phi^* \quad (21)$$

is a quadratic part of the action (including gauge fixing terms) and

$$S_I \equiv \frac{1}{4} \int d^4x \, d^4\theta \left(\phi^*(e^{2V} - 1)\phi + \tilde{\phi}^*(e^{-2V} - 1)\tilde{\phi} \right) \tag{22}$$

is the interaction.

Then, setting all fields $\phi_0 = 0$, generating functional (11) can be written as

$$Z[J, j]\Big|_{\phi_0=0} = \prod_i \left(\det PV\left(\frac{1}{i}\frac{\delta}{\delta J}, M_i\right) \right)^{c_i} \exp\left(iS_I\left[\frac{1}{i}\frac{\delta}{\delta J}, \frac{1}{i}\frac{\delta}{\delta j}\right] \right) \times$$
$$\times \int DV D\phi \exp\left(i\, S_2(V, \phi) + i\, S_S \right)\Big|_{\phi_0=0}, \tag{23}$$

where from all matter superfields we wrote explicitly only the field ϕ for the brevity of the notations. (Below we will also omit a condition $\phi_0 = 0$ for the same reasons.)

The remaining integral in this expression is gaussian and can be easily calculated:

$$Z[J, j] = \prod_i \left(\det PV\left(\frac{1}{i}\frac{\delta}{\delta J}, M_i\right) \right)^{c_i} \exp\left(iS_I\left[\frac{1}{i}\frac{\delta}{\delta J}, \frac{1}{i}\frac{\delta}{\delta j}\right] \right) Z_0, \tag{24}$$

where

$$Z_0 = \exp\left\{ i \int d^4x \, d^4\theta \left(-J\frac{e_0^2}{\partial^2\left(1 + \partial^{2n}/\Lambda^{2n}\right)}J + j^*\frac{1}{\partial^2 + m^2}j + \tilde{j}^*\frac{1}{\partial^2 + m^2}\tilde{j} + \right.\right.$$
$$\left.\left. + j\frac{m}{\partial^2 + m^2}\frac{D^2}{4\partial^2}\tilde{j} + j^*\frac{m}{\partial^2 + m^2}\frac{\bar{D}^2}{4\partial^2}\tilde{j}^* \right) \right\}. \tag{25}$$

Let us differentiate this expression with respect to J_x (an index x here and later denotes the argument of a function):

$$\frac{\delta Z}{\delta J_x} = -\prod_i \left(\det PV\left(\frac{1}{i}\frac{\delta}{\delta J}, M_i\right) \right)^{c_i} \exp\left(iS_I\left[\frac{1}{i}\frac{\delta}{\delta J}, \frac{1}{i}\frac{\delta}{\delta j}\right] \right) \frac{2ie_0^2}{\partial^2\left(1 + \partial^{2n}/\Lambda^{2n}\right)} J_x Z_0 =$$
$$= -\frac{2ie_0^2}{\partial^2\left(1 + \partial^{2n}/\Lambda^{2n}\right)} \left(J_x + \sum_i c_i \frac{\delta}{\delta V_x} \ln\det PV\left(\frac{1}{i}\frac{\delta}{\delta J}, M_i\right) + \frac{\delta S_I}{\delta V_x}\left[\frac{1}{i}\frac{\delta}{\delta J}, \frac{1}{i}\frac{\delta}{\delta j}\right] \right) Z, \tag{26}$$

where

$$\frac{\delta S_I}{\delta V_x}[V, \phi] = \frac{1}{2}\left(\phi_x^* e^{2V_x} \phi_x - \tilde{\phi}_x^* e^{-2V_x} \tilde{\phi}_x \right). \tag{27}$$

according to Eq. (22).

Taking into account that $Z = \exp(iW)$, we rewrite this identity in terms of the functional W as follows:

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$$\frac{\delta W}{\delta J_x} = -\frac{2e_0^2}{\partial^2\left(1 + \partial^{2n}/\Lambda^{2n}\right)}\left(J_x + \sum_i c_i \frac{\delta}{\delta V_x}\ln\det PV\Big(\frac{1}{i}\frac{\delta}{\delta J} + \frac{\delta W}{\delta J}, M_i\Big) + \right.$$
$$\left. +\frac{\delta S_I}{\delta V_x}\Big[\frac{1}{i}\frac{\delta}{\delta J} + \frac{\delta W}{\delta J}, \frac{1}{i}\frac{\delta}{\delta j} + \frac{\delta W}{\delta j}\Big]\right). \tag{28}$$

Passing at last to the effective action Γ, we obtain the following identity:

$$\frac{\delta\Gamma}{\delta V_x} = \frac{1}{2e_0^2}\partial^2\left(1 + \partial^{2n}/\Lambda^{2n}\right)V_x + \sum_i c_i \frac{\delta}{\delta V_x}\ln\det PV\Big(\frac{1}{i}\frac{\delta}{\delta J} + V, M_i\Big) +$$
$$+\frac{\delta S_I}{\delta V_x}\Big[\frac{1}{i}\frac{\delta}{\delta J} + V, \frac{1}{i}\frac{\delta}{\delta j} + \phi\Big]. \tag{29}$$

Here the derivatives with respect to the sources should be expressed in terms of derivatives with respect to the fields. For example, taking into account that

$$\frac{\delta\phi_x}{\delta\phi_y} = -\frac{1}{2}D^2\delta_{xy}^8; \qquad \frac{\delta\phi_x^*}{\delta\phi_y^*} = -\frac{1}{2}D^2\delta_{xy}^8 \tag{30}$$

where

$$\delta_{xy}^8 \equiv \delta^4(x - y)\delta^4(\theta_x - \theta_y), \tag{31}$$

we obtain

$$\frac{\delta}{\delta j_x^*} = \int d^8z\left(\frac{\delta\phi_z}{\delta j_x^*}\frac{D^2}{8\partial^2}\frac{\delta}{\delta\phi_z} + \frac{\delta\phi_z^*}{\delta j_x^*}\frac{D^2}{8\partial^2}\frac{\delta}{\delta\phi_z^*} + \frac{\delta\tilde\phi_z}{\delta j_x^*}\frac{D^2}{8\partial^2}\frac{\delta}{\delta\tilde\phi_z} + \frac{\delta\tilde\phi_z^*}{\delta j_x^*}\frac{D^2}{8\partial^2}\frac{\delta}{\delta\tilde\phi_z^*} + \right.$$
$$\left. +\frac{\delta V_z}{\delta j_x^*}\frac{\delta}{\delta V_z}\right) = \int d^8z\left[\left(\frac{\delta^2\Gamma}{\delta\phi_z\delta\phi_x^*}\right)^{-1}\frac{D^2}{8\partial^2}\frac{\delta}{\delta\phi_z} + \left(\frac{\delta^2\Gamma}{\delta\phi_z^*\delta\phi_x^*}\right)^{-1}\frac{D^2}{8\partial^2}\frac{\delta}{\delta\phi_z^*} + \right.$$
$$\left. +\left(\frac{\delta^2\Gamma}{\delta\tilde\phi_z\delta\phi_x^*}\right)^{-1}\frac{D^2}{8\partial^2}\frac{\delta}{\delta\tilde\phi_z} + \left(\frac{\delta^2\Gamma}{\delta\tilde\phi_z^*\delta\phi_x^*}\right)^{-1}\frac{D^2}{8\partial^2}\frac{\delta}{\delta\tilde\phi_z^*} + \left(\frac{\delta^2\Gamma}{\delta V_z\delta\phi_x^*}\right)^{-1}\frac{\delta}{\delta V_z}\right], \tag{32}$$

where

$$\int d^8z \equiv \int d^4z\, d^4\theta_z. \tag{33}$$

It is important to note, that if all fields are set equal to 0, nontrivial contributions come only from the first and the fourth terms due to the continuousness of matter superfields lines (if $m = 0$ only the first term gives a nontrivial contribution).

Let us substitute the explicit expression for S_I, given by Eq. (27), into Eq. (29), differentiate the result with respect to V_y and then set all fields equal to 0:

$$\frac{\delta^2 \Gamma}{\delta V_x \, \delta V_y}\bigg|_{V,\phi=0} = \frac{1}{2e_0^2}\partial^2\big(1+\partial^{2n}/\Lambda^{2n}\big)\delta_{xy}^8 + \sum_i c_i \frac{\delta^2}{\delta V_x \delta V_y}\ln\det PV\Big(\frac{1}{i}\frac{\delta}{\delta J}+V,M_i\Big) +$$

$$+\frac{\delta}{\delta V_y}\int d^8x_1 \Big(\frac{\delta^2\Gamma}{\delta\phi_x^*\delta\phi_1}\Big)^{-1}\frac{D_1^2}{16i\partial^2}\frac{\delta}{\delta\phi_1}\exp\Big(\frac{2}{i}\frac{\delta}{\delta J_x}+2V_x\Big)\phi_x +$$

$$+\frac{\delta}{\delta V_y}\int d^8x_1 \Big(\frac{\delta^2\Gamma}{\delta\phi_x^*\delta\tilde\phi_1^*}\Big)^{-1}\frac{\bar D_1^2}{16i\partial^2}\frac{\delta}{\delta\tilde\phi_1^*}\exp\Big(\frac{2}{i}\frac{\delta}{\delta J_x}+2V_x\Big)\phi_x -$$

$$-\frac{\delta}{\delta V_y}\int d^8x_1 \Big(\frac{\delta^2\Gamma}{\delta\tilde\phi_x^*\delta\tilde\phi_1}\Big)^{-1}\frac{D^2}{16i\partial^2}\frac{\delta}{\delta\tilde\phi_1}\exp\Big(-\frac{2}{i}\frac{\delta}{\delta J_x}-2V_x\Big)\tilde\phi_x -$$

$$-\frac{\delta}{\delta V_y}\int d^8x_1 \Big(\frac{\delta^2\Gamma}{\delta\tilde\phi_x^*\delta\phi_1^*}\Big)^{-1}\frac{\bar D^2}{16i\partial^2}\frac{\delta}{\delta\phi_1^*}\exp\Big(-\frac{2}{i}\frac{\delta}{\delta J_x}-2V_x\Big)\tilde\phi_x\bigg|_{V,\phi=0}. \tag{34}$$

Using the Leibnitz rule this expression can be presented in the following form:

$$\frac{\delta^2 \Gamma}{\delta V_x \, \delta V_y}\bigg|_{V,\phi=0} = \frac{1}{2e_0^2}\partial^2\big(1+\partial^{2n}/\Lambda^{2n}\big)\delta_{xy}^8 + \sum_i c_i \frac{\delta^2}{\delta V_x \delta V_y}\ln\det PV\Big(\frac{1}{i}\frac{\delta}{\delta J}+V,M_i\Big) +$$

$$+\int d^8x_1 \Big(\frac{\delta^2\Gamma}{\delta\phi_x^*\delta\phi_1}\Big)^{-1}\frac{D_1^2}{16i\partial^2}\frac{\delta}{\delta V_y}\frac{\delta}{\delta\phi_1}\exp\Big(\frac{2}{i}\frac{\delta}{\delta J_x}+2V_x\Big)\phi_x - \int d^8x_1\, d^8x_2\, d^8x_3\ \times$$

$$\times\Bigg\{\Big(\frac{\delta^2\Gamma}{\delta\phi_x^*\delta\phi_2}\Big)^{-1}\frac{D_2^2}{8\partial_2^2}\frac{\bar D_3^2}{8\partial_3^2}\frac{\delta^3\Gamma}{\delta V_y\delta\phi_2\delta\phi_3^*}\Big(\frac{\delta^2\Gamma}{\delta\phi_3^*\delta\phi_1}\Big)^{-1} + \Big(\frac{\delta^2\Gamma}{\delta\phi_x^*\delta\tilde\phi_2^*}\Big)^{-1}\frac{\bar D_2^2}{8\partial_2^2}\frac{D_3^2}{8\partial_3^2}\ \times$$

$$\times\frac{\delta^3\Gamma}{\delta V_y\delta\tilde\phi_2^*\delta\tilde\phi_3}\Big(\frac{\delta^2\Gamma}{\delta\tilde\phi_3\delta\phi_1}\Big)^{-1} + \Big(\frac{\delta^2\Gamma}{\delta\phi_x^*\delta\phi_2}\Big)^{-1}\frac{D_2^2}{8\partial_2^2}\frac{D_3^2}{8\partial_3^2}\frac{\delta^3\Gamma}{\delta V_y\delta\phi_2\delta\tilde\phi_3}\Big(\frac{\delta^2\Gamma}{\delta\tilde\phi_3\delta\phi_1}\Big)^{-1} +$$

$$+\Big(\frac{\delta^2\Gamma}{\delta\phi_x^*\delta\tilde\phi_2^*}\Big)^{-1}\frac{\bar D_2^2}{8\partial_2^2}\frac{\bar D_3^2}{8\partial_3^2}\frac{\delta^3\Gamma}{\delta V_y\delta\tilde\phi_2^*\delta\phi_3^*}\Big(\frac{\delta^2\Gamma}{\delta\phi_3^*\delta\phi_1}\Big)^{-1}\Bigg\}\frac{D_1^2}{16i\partial_1^2}\frac{\delta}{\delta\phi_1}\exp\Big(\frac{2}{i}\frac{\delta}{\delta J_x}+2V_x\Big)\phi_x +$$

$$+\text{ the other similar terms}\bigg|_{V,\phi=0} \tag{35}$$

This equation is a Schwinger-Dyson equation for the considered theory. It is more convenient to present it in the graphical form, as a sum of two diagrams, presented in Fig. 1. The terms, which contain the integration over d^8x_1 only, correspond to the second diagram in this figure, while the terms, containing integration over $d^8x_1\, d^8x_2\, d^8x_3$, – to the first diagram. Here the double lines denote the exact propagator, the large circle – the effective vertex and two adjacent circles – the effective vertex, consisting of 1PI diagrams, in which one of the external lines is attached to the very left edge.

Contributions of the Pauli-Villars determinants are calculated similarly. It is convenient to introduce the notation

$$Z_{PV}[V,j_{PV},M_i] \equiv \int D\Phi\, D\tilde\Phi \exp\Big(iS_{PV}+iS_{S_{PV}}+iS_{\Phi_0}\Big), \tag{36}$$

where

$$S_{S_{PV}} = \int d^4x \, d^2\theta \left(j_{PV} \, \Phi + \tilde{j}_{PV} \, \tilde{\Phi} \right) + \int d^4x \, d^2\bar{\theta} \left(j_{PV}^* \Phi^* + \tilde{j}_{PV}^* \tilde{\Phi}^* \right);$$

$$S_{\Phi_0} = \frac{1}{4} \int d^4x \, d^4\theta \left(\Phi_0^* \, e^{2V} \Phi + \Phi^* \, e^{2V} \Phi_0 + \tilde{\Phi}_0^* \, e^{-2V} \tilde{\Phi} + \tilde{\Phi}^* \, e^{-2V} \tilde{\Phi}_0 \right). \qquad (37)$$

Then according to Eq. (12) the determinants, which is necessary for us, can be written as

$$\left(\det PV(V, M_i) \right)^{-1} = Z_{PV}[V, j_{PV}, M_i] \Big|_{\Phi_0, j_{PV}=0}. \qquad (38)$$

Moreover, let us define functionals

$$W_{PV}[V, j_{PV}, M_i] \equiv -i \ln Z_{PV}[V, j_{PV}, M_i];$$

$$\Gamma_{PV}[V, \Phi, M_i] = W_{PV} - \int d^4x \, d^2\theta \left(j_{PV} \, \Phi + \tilde{j}_{PV} \, \tilde{\Phi} \right) -$$

$$- \int d^4x \, d^2\bar{\theta} \left(j_{PV}^* \Phi^* + \tilde{j}_{PV}^* \tilde{\Phi}^* \right), \qquad (39)$$

where the fields Φ and $\tilde{\Phi}$ are related with the sources j_{PV} and $\tilde{j}_{PV}$ by equations

$$\Phi = \frac{\delta W_{PV}}{\delta j_{PV}}; \qquad \tilde{\Phi} = \frac{\delta W_{PV}}{\delta \tilde{j}_{PV}}. \qquad (40)$$

Differentiating Eq. (36) with respect to V_x we find

$$\frac{\delta Z_{PV}}{\delta V_x} = -\frac{i}{2} \left(\frac{\delta}{\delta j_{x\,PV}^*} e^{2V_x} \frac{\delta}{\delta j_{x\,PV}} - \frac{\delta}{\delta \tilde{j}_{x\,PV}^*} e^{-2V_x} \frac{\delta}{\delta \tilde{j}_{x\,PV}} \right) Z_{PV}. \qquad (41)$$

Dividing this expression by Z_{PV} and setting Φ and $\tilde{\Phi}$ equal to 0, we obtain

$$\frac{\delta W_{PV}}{\delta V_x} \Big|_{\Phi=0} = -\frac{1}{2} \frac{\delta}{\delta j_{x\,PV}^*} e^{2V_x} \frac{\delta W_{PV}}{\delta j_{x\,PV}} + \frac{1}{2} \frac{\delta}{\delta \tilde{j}_{x\,PV}^*} e^{-2V_x} \frac{\delta W_{PV}}{\delta \tilde{j}_{x\,PV}} \Big|_{\Phi=0}. \qquad (42)$$

In terms of Γ_{PV} this identity can be written as

$$\frac{\delta \Gamma_{PV}}{\delta V_x} \Big|_{\Phi=0} = -\frac{1}{2} \frac{\delta}{\delta j_{x\,PV}^*} e^{2V_x} \Phi_x + \frac{1}{2} \frac{\delta}{\delta \tilde{j}_{x\,PV}^*} e^{-2V_x} \tilde{\Phi}_x \Big|_{\Phi=0}. \qquad (43)$$

Passing in this equation from the derivatives with respect to the sources to the derivatives with respect to the fields we find, that the expression for

$$\frac{\delta^2}{\delta V_x \delta V_y} \ln \det PV \left(\frac{1}{i} \frac{\delta}{\delta J} + V, M_i \right) \Big|_{V, \Phi=0} \qquad (44)$$

coincides with the last four terms in Eq. (34) up to the substitutions $\phi \to \Phi$, $\tilde{\phi} \to \tilde{\Phi}$, $\phi^* \to \Phi^*$ and $\tilde{\phi}^* \to \tilde{\Phi}^*$. Thus we conclude, that the structure of terms, coming from

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the Pauli-Villars determinants, is actually the same as the structure of the other terms and their calculation is made similarly.

We also need Schwinger-Dyson equations of another type. In order to derive them let us differentiate the generating functional with respect to ϕ_0^*. Then

$$\frac{\delta Z}{\delta \phi_{0z}^*} = \frac{1}{4} \exp\left(\frac{2}{i}\frac{\delta}{\delta J_z}\right)\frac{\delta Z}{\delta j_z}. \tag{45}$$

Dividing this equation by Z, we find

$$\frac{\delta W}{\delta \phi_{0z}^*} = \frac{1}{4} \exp\left(\frac{2}{i}\frac{\delta}{\delta J_z} + 2\frac{\delta W}{\delta J_z}\right)\frac{\delta W}{\delta j_z}. \tag{46}$$

Because the field ϕ_0^* is a parameter of the effective action, we obtain the identity

$$\frac{\delta \Gamma}{\delta \phi_{0z}^*} = \frac{\delta W}{\delta \phi_{0z}^*} = \frac{1}{4} \exp\left(\frac{2}{i}\frac{\delta}{\delta J_z} + 2V_z\right)\phi_z. \tag{47}$$

Differentiating this equation and setting then all fields equal to 0, we find the following equalities, which will be useful for us later:

$$\frac{\delta^2 \Gamma}{\delta \phi_y \delta \phi_{0z}^*} = \frac{1}{4}\frac{\delta}{\delta \phi_y} \exp\left(\frac{2}{i}\frac{\delta}{\delta J_z} + 2V_z\right)\phi_z;$$

$$\frac{\delta^2 \Gamma}{\delta \tilde{\phi}_y^* \delta \phi_{0z}^*} = \frac{1}{4}\frac{\delta}{\delta \tilde{\phi}_y^*} \exp\left(\frac{2}{i}\frac{\delta}{\delta J_z} + 2V_z\right)\phi_z;$$

$$\frac{\delta^3 \Gamma}{\delta V_x \delta \phi_y \delta \phi_{0z}^*} = \frac{1}{4}\frac{\delta}{\delta V_x}\frac{\delta}{\delta \phi_y} \exp\left(\frac{2}{i}\frac{\delta}{\delta J_z} + 2V_z\right)\phi_z;$$

$$\frac{\delta^3 \Gamma}{\delta V_x \delta \tilde{\phi}_y^* \delta \phi_{0z}^*} = \frac{1}{4}\frac{\delta}{\delta V_x}\frac{\delta}{\delta \tilde{\phi}_y^*} \exp\left(\frac{2}{i}\frac{\delta}{\delta J_z} + 2V_z\right)\phi_z. \tag{48}$$

Let us relate these expressions with the ordinary Green functions. For this purpose it is possible to use other Schwinger-Dyson equations, which can be derived similarly to those, which were described earlier (differentiating Z with respect to j^* and $\tilde{j}$):

$$(\partial^2 + m^2)\phi_x^* = \frac{D_x^2}{2}\left(\frac{\delta \Gamma}{\delta \phi_x} + \frac{\bar{D}_x^2}{8}\left(\left[\exp\left(\frac{2}{i}\frac{\delta}{\delta J_x} + 2V_x\right) - 1\right]\phi_x^*\right)\right) +$$

$$+2m\left(\frac{\delta \Gamma}{\delta \tilde{\phi}_x^*} + \frac{D_x^2}{8}\left(\left[\exp\left(-\frac{2}{i}\frac{\delta}{\delta J_x} - 2V_x\right) - 1\right]\tilde{\phi}_x\right)\right); \tag{49}$$

$$(\partial^2 + m^2)\tilde{\phi}_x = \frac{\bar{D}_x^2}{2}\left(\frac{\delta \Gamma}{\delta \tilde{\phi}_x^*} + \frac{D_x^2}{8}\left(\left[\exp\left(-\frac{2}{i}\frac{\delta}{\delta J_x} - 2V_x\right) - 1\right]\tilde{\phi}_x\right)\right) +$$

$$+2m\left(\frac{\delta \Gamma}{\delta \phi_x} + \frac{\bar{D}_x^2}{8}\left(\left[\exp\left(\frac{2}{i}\frac{\delta}{\delta J_x} + 2V_x\right) - 1\right]\phi_x^*\right)\right). \tag{50}$$

Differentiating Eqs. (49) and (50) with respect to ϕ_y^*, and then setting all fields equal to 0, we obtain a system of equations

$$-m^2 D_x^2 \delta_{xy}^8 = D_x^2 \left(\frac{\delta^2 \Gamma}{\delta \phi_y^* \delta \phi_x} + \frac{\delta}{\delta \phi_y^*} \frac{\bar{D}_x^2}{8} \exp\left(\frac{2}{i} \frac{\delta}{\delta J_x} + 2V_x \right) \phi_x^* \right) +$$
$$+ 4m \left(\frac{\delta^2 \Gamma}{\delta \phi_y^* \delta \tilde{\phi}_x^*} + \frac{D_x^2}{8} \frac{\delta}{\delta \phi_y^*} \exp\left(-\frac{2}{i} \frac{\delta}{\delta J_x} - 2V_x \right) \tilde{\phi}_x \right); \quad (51)$$

$$0 = \bar{D}_x^2 \left(\frac{\delta^2 \Gamma}{\delta \phi_y^* \delta \tilde{\phi}_x^*} + \frac{\delta}{\delta \phi_y^*} \frac{D_x^2}{8} \exp\left(-\frac{2}{i} \frac{\delta}{\delta J_x} - 2V_x \right) \tilde{\phi}_x \right) +$$
$$+ 4m \left(\frac{\delta^2 \Gamma}{\delta \phi_y^* \delta \phi_x} + \frac{\bar{D}_x^2}{8} \frac{\delta}{\delta \phi_y^*} \exp\left(\frac{2}{i} \frac{\delta}{\delta J_x} + 2V_x \right) \phi_x^* \right). \quad (52)$$

After simple transformations from Eqs. (51) and (52) we find

$$\frac{\delta}{\delta \phi_1} \exp\left(\frac{2}{i} \frac{\delta}{\delta J_x} + 2V_x \right) \phi_x = \frac{\bar{D}_x^2}{2\partial^2} \frac{\delta^2 \Gamma}{\delta \phi_1 \delta \phi_x^*};$$
$$\frac{\delta}{\delta \tilde{\phi}_1^*} \exp\left(\frac{2}{i} \frac{\delta}{\delta J_x} + 2V_x \right) \phi_x = \frac{\bar{D}_x^2}{2\partial^2} \left(\frac{\delta^2 \Gamma}{\delta \tilde{\phi}_1^* \delta \phi_x^*} + \frac{m}{4} D_x^2 \delta_{1x}^8 \right). \quad (53)$$

Taking into account that a variational derivative with respect to a chiral superfield is a chiral superfield again, and using dimensional arguments, we find, that the explicit form of the two-point Green functions for the matter superfields is

$$\frac{\delta^2 \Gamma}{\delta \phi_x^* \delta \phi_y} = \frac{D_x^2 \bar{D}_x^2}{16} G(\partial^2) \delta_{xy}^8; \qquad \frac{\delta^2 \Gamma}{\delta \phi_x \delta \tilde{\phi}_y} = -\frac{\bar{D}_x^2}{4} m J(\partial^2) \delta_{xy}^8. \quad (54)$$

Note, that the corresponding inverse functions, which are determined from equations

$$\int d^8 y \left(\frac{\delta^2 \Gamma}{\delta \phi_x^* \delta \phi_y} \right)^{-1} \frac{D_y^2}{8\partial^2} \frac{\delta^2 \Gamma}{\delta \phi_y \delta \phi_z^*} + \int d^8 y \left(\frac{\delta^2 \Gamma}{\delta \phi_x^* \delta \tilde{\phi}_y^*} \right)^{-1} \frac{\bar{D}_y^2}{8\partial^2} \frac{\delta^2 \Gamma}{\delta \tilde{\phi}_y^* \delta \phi_z^*} = -\frac{D_x^2}{2} \delta_{xz}^8;$$
$$\int d^8 y \left(\frac{\delta^2 \Gamma}{\delta \phi_x^* \delta \phi_y} \right)^{-1} \frac{D_y^2}{8\partial^2} \frac{\delta^2 \Gamma}{\delta \phi_y \delta \tilde{\phi}_z} + \int d^8 y \left(\frac{\delta^2 \Gamma}{\delta \phi_x^* \delta \tilde{\phi}_y^*} \right)^{-1} \frac{\bar{D}_y^2}{8\partial^2} \frac{\delta^2 \Gamma}{\delta \tilde{\phi}_y^* \delta \tilde{\phi}_z} = 0, \quad (55)$$

are

$$\left(\frac{\delta^2 \Gamma}{\delta \phi_x^* \delta \phi_y} \right)^{-1} = -\frac{G D_x^2 \bar{D}_x^2}{4(\partial^2 G^2 + m^2 J^2)} \delta_{xy}^8; \qquad \left(\frac{\delta^2 \Gamma}{\delta \phi_x \delta \tilde{\phi}_y} \right)^{-1} = -\frac{m J \bar{D}_x^2}{\partial^2 G^2 + m^2 J^2} \delta_{xy}^8. \quad (56)$$

Nevertheless, there are also the following expressions

$$\frac{\delta}{\delta V_y} \frac{\delta}{\delta \phi_1} \exp\left(\frac{2}{i} \frac{\delta}{\delta J_x} + 2V_x \right) \phi_x; \qquad \frac{\delta}{\delta V_y} \frac{\delta}{\delta \tilde{\phi}_1^*} \exp\left(\frac{2}{i} \frac{\delta}{\delta J_x} + 2V_x \right) \phi_x. \quad (57)$$

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in Eq. (35). In order to calculate them at zero momentum of the gauge field V, we need supersymmetric Ward identities.

It is necessary to note, that from Schwinger-Dyson equations (49) and (50) similarly to identities (53) it is easy to find

$$-\frac{D_x^2}{2}\frac{\delta^2\Gamma}{\delta V_y\delta\phi_1\delta\phi_{0x}^*} = -\frac{D_x^2}{8}\frac{\delta}{\delta V_y}\frac{\delta}{\delta\phi_1}\exp\left(\frac{2}{i}\frac{\delta}{\delta J_x}+2V_x\right)\phi_x = \frac{\delta^3\Gamma}{\delta V_y\delta\phi_1\delta\phi_x^*}; \quad (58)$$

$$-\frac{D_x^2}{2}\frac{\delta^2\Gamma}{\delta V_y\delta\tilde{\phi}_1^*\delta\phi_{0x}^*} = -\frac{D_x^2}{8}\frac{\delta}{\delta V_y}\frac{\delta}{\delta\tilde{\phi}_1^*}\exp\left(\frac{2}{i}\frac{\delta}{\delta J_x}+2V_x\right)\phi_x = \frac{\delta^3\Gamma}{\delta V_y\delta\tilde{\phi}_1^*\delta\phi_x^*}. \quad (59)$$

4   Ward identities for $N=1$ supersymmetric electrodynamics

The Ward identities for the supersymmetric electrodynamics are obtained by the standard way [6]: substitution (9) is performed in generating functional (11), the result is differentiated with respect to A in the limit $A \to 0$, and then we pass from the generating functional Z to the functional Γ. As a result of this operation we obtain the identity

$$\left(\bar{D}_x^2 + D_x^2\right)\left(\frac{\delta\Gamma}{\delta V_x} + \frac{1}{2e_0^2}\partial^2\left(1+\frac{\partial^{2n}}{\Lambda^{2n}}\right)V_x\right) - \bar{D}_x^2\left(2\phi_0\frac{\delta\Gamma}{\delta\phi_0} - 2\tilde{\phi}_0\frac{\delta\Gamma}{\delta\tilde{\phi}_0} + \frac{1}{4}\phi_x\frac{D_x^2}{\partial^2}\frac{\delta\Gamma}{\delta\phi_x} - \right.$$
$$\left.-\frac{1}{4}\tilde{\phi}_x\frac{D_x^2}{\partial^2}\frac{\delta\Gamma}{\delta\tilde{\phi}_x}\right) - D_x^2\left(2\phi_0^*\frac{\delta\Gamma}{\delta\phi_0^*} - 2\tilde{\phi}_0^*\frac{\delta\Gamma}{\delta\tilde{\phi}_0^*} + \frac{1}{4}\phi_x^*\frac{\bar{D}_x^2}{\partial^2}\frac{\delta\Gamma}{\delta\phi_x^*} - \frac{1}{4}\tilde{\phi}_x^*\frac{\bar{D}_x^2}{\partial^2}\frac{\delta\Gamma}{\delta\tilde{\phi}_x^*}\right) = 0. \quad (60)$$

(in which the fields are not yet set equal to 0). Differentiating this equality with respect to ϕ_y and ϕ_{0z}^* and then setting all fields equal to 0, we find the Ward identity for the three-point Green function

$$(\bar{D}_x^2 + D_x^2)\frac{\delta^3\Gamma}{\delta V_x\delta\phi_y\delta\phi_{0z}^*} = 2\bar{D}_x^2\delta_{xy}^8\frac{\delta^2\Gamma}{\delta\phi_x\delta\phi_{0z}^*} + 2D_x^2\left(\delta_{xz}^8\frac{\delta^2\Gamma}{\delta\phi_y\delta\phi_{0x}^*}\right) \quad (61)$$

where we take into account that a variational derivative with respect to a chiral superfield is a chiral superfield again.

Solution of Ward identity (61) in the limit $p \to 0$ is a function

$$\frac{\delta^3\Gamma}{\delta V_x\delta\phi_y\delta\phi_{0z}^*}\bigg|_{p=0} = \frac{1}{4}\frac{\delta}{\delta V_x}\frac{\delta}{\delta\phi_y}\exp\left(\frac{2}{i}\frac{\delta}{\delta J_z}+2V_z\right)\phi_z\bigg|_{p=0} =$$

$$= -2\partial^2\Pi_{1/2x}\left(\bar{D}_x^2\delta_{xy}^8\delta_{xz}^8\right)F(q^2) + \frac{1}{8}D^aC_{ab}\bar{D}_x^2\left(\bar{D}_x^2\delta_{xy}^8D_x^b\delta_{xz}^8\right)f(q^2) +$$

$$-\frac{1}{16}q^\mu G'(q^2)\bar{D}\gamma^\mu\gamma_5 D_x\left(\bar{D}_x^2\delta_{xy}^8\delta_{xz}^8\right) - \frac{1}{4}\bar{D}_x^2\delta_{xy}^8\delta_{xz}^8\,G(q^2), \quad (62)$$

where

$$\Pi_{1/2} = -\frac{1}{16\partial^2} D^a \bar{D}^2 C_{ab} D^b = -\frac{1}{16\partial^2} D^a D^2 C_{ab} D^b \qquad (63)$$

is the supersymmetric transversal projector. $F(q^2)$ and $f(q^2)$ are some functions of the matter field momentum q, which can not be determined from the Ward identity. Using Eq. (58) we also obtain

$$\frac{\delta^3\Gamma}{\delta V_x \delta\phi_y \delta\phi_z^*}\bigg|_{p=0} = \partial^2 \Pi_{1/2x} \Big(\bar{D}_x^2 \delta_{xy}^8 D_x^2 \delta_{xz}^8\Big) F(q^2) +$$

$$+\frac{1}{32} q^\mu G'(q^2) \bar{D} \gamma^\mu \gamma_5 D_x \Big(\bar{D}_x^2 \delta_{xy}^8 D_x^2 \delta_{xz}^8\Big) + \frac{1}{8} \bar{D}_x^2 \delta_{xy}^8 D_x^2 \delta_{xz}^8 \, G(q^2), \quad (64)$$

Similarly, differentiating Eq. (60) with respect to ϕ_y and $\tilde{\phi}_{0z}$, we obtain Ward identity

$$(D_x^2 + \bar{D}_x^2) \frac{\delta^3\Gamma}{\delta V_x \delta\phi_y \delta\tilde{\phi}_{0z}} = -\bar{D}_x^2 \bigg(\bar{D}_x^2 \delta_{xy}^8 \frac{D_x^2}{8\partial^2} \frac{\delta^2\Gamma}{\delta\phi_x \delta\tilde{\phi}_{0z}} + 2\delta_{xz}^8 \frac{\delta^2\Gamma}{\delta\tilde{\phi}_{0x}\delta\phi_y}\bigg) \qquad (65)$$

Taking into account that according to Eqs. (53) and (54)

$$\frac{\delta^2\Gamma}{\delta\tilde{\phi}_{0x}\delta\phi_y} = \frac{m}{32\partial^2}\Big(J(\partial^2) - 1\Big)\bar{D}_y^2 D_y^2 \delta_{xy}^8, \qquad (66)$$

it is possible to rewrite the considered Ward identity in the following form:

$$(D_x^2 + \bar{D}_x^2)\frac{\delta^3\Gamma}{\delta V_x \delta\phi_y \delta\tilde{\phi}_{0z}} =$$

$$= \frac{m}{16}\bigg[\bar{D}_x^2 \delta_{xy}^8 \frac{D_x^2 D_x^2}{\partial^2}\Big(J(\partial^2) - 1\Big)\delta_{xz}^8 - \bar{D}_x^2\bigg(\delta_{xz}^8 \frac{D_x^2 \bar{D}_x^2}{\partial^2}\Big(J(\partial^2) - 1\Big)\delta_{xy}^8\bigg)\bigg]. \quad (67)$$

Its solution at $p = 0$ can be written as

$$\frac{\delta^3\Gamma}{\delta V_x \delta\phi_y \delta\tilde{\phi}_{0z}}\bigg|_{p=0} = \frac{1}{4}\frac{\delta}{\delta V_x}\frac{\delta}{\delta\phi_y}\exp\bigg(\frac{2}{i}\frac{\delta}{\delta J_z} + 2V_z\bigg)\tilde{\phi}_z^*\bigg|_{p=0} =$$

$$= -\frac{m}{2}\partial^2 \Pi_{1/2x}\bigg(\bar{D}_x^2 \delta_{xy}^8 D_x^2 \delta_{xz}^8 - D_x^2 \bar{D}_x^2 \delta_{xy}^8 \delta_{xz}^8\bigg)H(q^2) + \frac{m}{32} D^a C_{ab} D_x^2 \times$$

$$\times \Big(D_x^2 \bar{D}_x^2 \delta_{xy}^8 D_x^b \delta_{xz}^8\Big)h(q^2) + \frac{m}{16} J'(q^2)\bigg(\bar{D}_x^2 \delta_{xy}^8 D_x^2 \delta_{xz}^8 - D_x^2 \bar{D}_x^2 \delta_{xy}^8 \delta_{xz}^8\bigg) +$$

$$+\frac{m}{16}\bigg(\frac{J'(q^2)}{q^2} - \frac{J(q^2) - 1}{q^4}\bigg)\bigg(D_x^2 \bar{D}_x^2 \delta_{xy}^8 \frac{\bar{D}_x^2 D_x^2}{16}\delta_{xz}^8 + D_x^2 \bar{D}_x^2 \delta_{xy}^8 q^2 \delta_{xz}^8\bigg), \qquad (68)$$

where $H(q^2)$ and $h(q^2)$ are two more functions, which can not be found from the Ward identity. Here we take into account, that the result should be chiral in y and

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antisymmetric with respect to the replacement $y \leftrightarrow z$ after applying the operator D_z^2. Really, in this case from Eq. (59) we also obtain

$$(D_x^2 + \bar{D}_x^2)\frac{\delta^3\Gamma}{\delta V_x \delta \phi_y \delta \tilde{\phi}_z} = \frac{m}{2}\left[\bar{D}_x^2 \delta_{xy}^8 \bar{D}_x^2 J(\partial^2)\delta_{xz}^8 - \bar{D}_x^2 \delta_{xz}^8 \bar{D}_x^2 J(\partial^2)\delta_{xy}^8\right]. \tag{69}$$

Solution of this equation in the limit $p \to 0$ is

$$\frac{\delta^3\Gamma}{\delta V_x \delta \phi_y \delta \tilde{\phi}_z}\bigg|_{p=0} = -\frac{\bar{D}_z^2}{2}\frac{\delta^3\Gamma}{\delta V_x \delta \phi_y \delta \tilde{\phi}_{0z}}\bigg|_{p=0} =$$

$$= \frac{m}{4}\partial^2 \Pi_{1/2x}\left(\bar{D}_x^2 \delta_{xy}^8 D_x^2 \bar{D}_x^2 \delta_{xz}^8 - D_x^2 \bar{D}_x^2 \delta_{xy}^8 \bar{D}_x^2 \delta_{xz}^8\right)H(q^2) -$$

$$-\frac{m}{32}J'(q^2)\left(\bar{D}_x^2 \delta_{xy}^8 D_x^2 \bar{D}_x^2 \delta_{xz}^8 - D_x^2 \bar{D}_x^2 \delta_{xy}^8 \bar{D}_x^2 \delta_{xz}^8\right). \tag{70}$$

5 Two-point Green function of the gauge field

Let us now substitute the expressions, obtained above, into two-point Green function of the gauge field (35). Let us remind once again, that at first we make the calculation ignoring diagrams, containing insertions of counterterms on lines of the matter superfields. Then due to the supersymmetric gauge invariance quantum corrections to the effective action, corresponding to the two-point Green function of the gauge field, can be written as

$$\Gamma_V^{(2)} = -\frac{1}{16\pi}\int \frac{d^4p}{(2\pi)^4}\, V(-p)\, \partial^2 \Pi_{1/2} V(p)\, d_0^{-1}(\alpha_0, \Lambda/p). \tag{71}$$

where d_0 is a function, which can be determined from Eq. (35). Really, it is easy to see, that

$$\Pi_{1/2}\int d^4x\, d^4y\, \frac{\delta^2\Gamma}{\delta V_x\, \delta V_y}\bigg|_{V,\phi=0}\exp\left(ip_\mu x^\mu + iq_\mu y^\mu\right) =$$

$$= \frac{1}{8\pi}(2\pi)^4 \delta^4\left(p + q\right) p^2 \Pi_{1/2}\delta^4(\theta_x - \theta_y)\, d_0^{-1}(\alpha_0, \Lambda/p). \tag{72}$$

In order to find the function d_0 first we differentiate expression (35) with respect to $\ln\Lambda$, and then set $p = 0$ in it. (We must take into account the dependence $e_0 = e_0(\Lambda)$, which originates from diagrams with insertions of the counterterms on lines of the gauge superfield V.) Note, that due to the using of the higher derivative regularization we can directly differentiate the integrand. The differentiation with respect to $\ln\Lambda$ is needed in order to obtain a finite expression at $p = 0$. (Existence of the finite limit at $p \to 0$ will be proven later.)

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After differentiating Eq. (35) with respect to $\ln \Lambda$ the contribution of the two-point Green function of the gauge field to the effective action (without gauge fixing terms) can be written as

$$\frac{d}{d\ln\Lambda}\Gamma_V^{(2)} = \frac{1}{2}\frac{d}{d\ln\Lambda}\int d^8x\, d^8y\, V_x V_y \left(\left(T_{xy}^{(1)} + T_{xy}^{(2)} \right) - \sum_i c_i \left(T_{xy}^{(1)PV}(M_i) + T_{xy}^{(2)PV}(M_i) \right) \right),$$
(73)

where $T_{xy}^{(1)}$ denotes the sum of terms, which correspond to the first diagram in Fig. 1, and $T_{xy}^{(2)}$ is a sum of diagrams, corresponding to the second diagram in this figure. $T_{xy}^{(1)PV}(M_i)$ and $T_{xy}^{(2)PV}(M_i)$ in Eq. (73) are the similar contributions of diagrams with the Pauli-Villars fields.

After substitution vertex functions from Eqs. (62), (64), (68) and (70) and propagators from Eqs. (54), Weak rotation and some simple transformations, using the algebra of the covariant derivatives, we find, that in the momentum representation the first diagram is

$$\frac{1}{2}\frac{d}{d\ln\Lambda}\int d^8x\, d^8y\, V_x V_y \left(T_{xy}^{(1)} - \sum_i c_i T_{xy}^{(1)PV}(M_i) \right) =$$
$$= \frac{d}{d\ln\Lambda}\int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\left\{ V\partial^2\Pi_{1/2}V\left[\frac{8GF}{q^2G^2+m^2J^2} - \frac{m^2JJ'}{2q^2(q^2G^2+m^2J^2)} + \right.\right.$$
$$\left.\left. + \frac{1}{2}\frac{d}{dq^2}\left(\ln\left(q^2G^2+m^2J^2\right) + \frac{m^2J}{q^2G^2+m^2J^2} \right) \right] + V^2\frac{G^2}{q^2G^2+m^2J^2} \right\},$$
(74)

while the second one is

$$\frac{1}{2}\frac{d}{d\ln\Lambda}\int d^8x\, d^8y\, V_x V_y \left(T_{xy}^{(2)} - \sum_i c_i T_{xy}^{(2)PV}(M_i) \right) =$$
$$= \frac{d}{d\ln\Lambda}\int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\left\{ V\partial^2\Pi_{1/2}V\left[-\frac{8GF}{q^2G^2+m^2J^2} - \frac{m^2JJ'}{2q^2(q^2G^2+m^2J^2)} + \right.\right.$$
$$\left.\left. + \frac{m^2J(J-1)}{2q^4(q^2G^2+m^2J^2)} - \frac{8Gf+8m^2Jh}{q^2G^2+m^2J^2} \right] - V^2\frac{G^2}{q^2G^2+m^2J^2} \right\}.$$
(75)

Adding the results, we find

$$\frac{d}{d\ln\Lambda}\Gamma_V^{(2)} = \int \frac{d^4p}{(2\pi)^4}V(-p)\,\partial^2\Pi_{1/2}V(p)\,I,$$
(76)

where I denotes the following Euclidean integral:

$$I = \frac{d}{d\ln\Lambda}\int \frac{d^4q}{(2\pi)^4}\left\{ \frac{1}{2q^2}\frac{d}{dq^2}\left(\ln\left(q^2G^2+m^2J^2\right) + \frac{m^2J}{q^2G^2+m^2J^2} \right) - \right.$$

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$$-\frac{m^2 J \left(2q^2 J' - (J-1)\right)}{2q^4 \left(q^2 G^2 + m^2 J^2\right)} - \frac{8Gf + 8m^2 Jh}{q^2 G^2 + m^2 J^2} -$$

$$-\sum_i c_i \frac{1}{2q^2} \frac{d}{dq^2} \left(\ln \left(q^2 G_{PV}^2 + M_i^2 J_{PV}^2\right) + \frac{M_i^2 J_{PV}}{q^2 G_{PV}^2 + M_i^2 J_{PV}^2} \right) +$$

$$+ \sum_i c_i \left(\frac{M_i^2 J \left(2q^2 J' - (J-1)\right)}{2q^4 \left(q^2 G^2 + M_i^2 J^2\right)} + \frac{8Gf + 8M_i^2 Jh}{q^2 G^2 + M_i^2 J^2} \right) \Bigg\}. \qquad (77)$$

Thus we see, that all noninvariant terms, proportional to V^2, and also terms, containing the undefined functions F and H, are completely cancelled. Nevertheless the final result contains arbitrary functions f and h, which can not be found from the Ward identities. (In the two-loop approximation these functions are equal to 0. They are nontrivial only at the three loops.)

From Eqs. (74) and (75) we see, that for the first diagram in Fig. 1 the result is obtained exactly to all orders of the perturbation theory. In the three-loop approximation it was checked explicitly. The expression for the second diagram contains the arbitrary functions f and h. However, these functions can be possibly determined from other arguments, because the explicit three-loop calculations, made in [22], enable us to propose the following structure of the result:

$$I = \frac{d}{d\ln\Lambda} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{2q^2} \frac{d}{dq^2} \Bigg\{ \left(\ln \left(q^2 G^2 + m^2 J^2\right) + X(q^2, m^2) \right) -$$

$$- \sum_i c_i \left(\ln \left(q^2 G_{PV}^2 + M_i^2 J_{PV}^2\right) + X(q^2, M_i^2) \right) \Bigg\}, \qquad (78)$$

where $X(q^2, m^2)$ is a function, which is finite and mass independent constant at $q = 0$, and is equal to 0 in the limit $q \to \infty$. If this proposal is true, the integral over the momentum q is reduced to the integral from a total derivative and can be taken analytically. It will be made in the next section.

6 Renormgroup functions in N=1 supersymmetric electrodynamics.

Let us believe, that proposal (78) is true and find its consequences. For this purpose let us pass to the four-dimensional spherical coordinates and take the integral:

$$I = \frac{1}{32\pi^2} \frac{d}{d\ln\Lambda} \Bigg\{ \left(\ln \left(q^2 G^2 + m^2 J^2\right) + X(q^2, m^2) \right) -$$

$$- \sum_i c_i \left(\ln \left(q^2 G_{PV}^2 + M_i^2 J_{PV}^2\right) + X(q^2, M_i^2) \right) \Bigg\} \Bigg|_0^\infty. \qquad (79)$$

Due to the higher derivative regularization the functions $G(q)$, $J(q)$, $G_{PV}(q)$ and $J_{PV}(q)$ are equal to 1 in the limit $q \to \infty$, because all quantum corrections to the

classical values disappear in this limit. Really, with the higher derivatives all integrals are convergent at finite Λ and the momentum q is present in denominators of integrals, defining quantum corrections. For example,

$$\lim_{q \to \infty} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{\left(1 + k^{2n}/\Lambda^{2n}\right) k^2 (k+q)^2} = 0. \tag{80}$$

Therefore, taking into account that $\sum\limits_i c_i = 1$, the substitution at $q \to \infty$ in Eq. (79) is equal to 0.

Below we will consider massive and massless cases separately.

1. Massless case:

Taking into account, that the function J_{PV} is a finite, mass and Λ independent constant in the limit $q \to 0$, because it is defined by convergent (even in the limit $\Lambda \to \infty$), dimensionless integrals, which do not contain infrared divergences, we obtain

$$I = -\frac{d}{d \ln \Lambda} \frac{1}{32\pi^2} \left\{ \ln G^2 - \sum_i c_i \ln \left(M_i^2 J_{PV}^2 \right) \right\} \Bigg|_{q=0} = \frac{1}{16\pi^2} \left(1 - \frac{d \ln G}{d \ln \Lambda} \right) \Bigg|_{q=0}. \tag{81}$$

Now let us prove, that such limit exists, because the result in principle can diverge at $q \to 0$. First we will suppose, that all terms, which gives 0 in the limit $\Lambda \to \infty$, are removed from the function $G(\alpha_0, q/\Lambda)$. (For example, the terms, proportional to $(q/\Lambda)^k$, where k is a positive constant.) It is evident, that removing of such terms does not change renormgroup functions. Because in this case ZG is finite due to the definition of the renormalization constant Z and does not depend on Λ, then

$$\Lambda \frac{d}{d\Lambda} \ln G\left(\alpha_0(\alpha, \Lambda/\mu), \Lambda/p\right) \Bigg|_{p=0} =$$

$$= -\Lambda \frac{d}{d\Lambda} \ln Z\left(\alpha_0(\alpha, \Lambda/\mu), \Lambda/p\right) \Bigg|_{p=0} = -\gamma\left(\alpha_0(\alpha, \Lambda/\mu)\right). \tag{82}$$

The anomalous dimension $\gamma(\alpha_0)$ is finite at $\Lambda < \infty$, so that limit (81) exists in all orders of the perturbation theory.

From Eqs. (71), (76) and (81) we obtain

$$\frac{d}{d \ln \Lambda} d_0^{-1}\left(\alpha_0, \Lambda/p\right) \Bigg|_{p=0} = \frac{1}{\pi} \frac{d}{d \ln \Lambda} \left(\ln \frac{\Lambda}{p} - G\left(\alpha_0, \Lambda/p\right) \right) \Bigg|_{p=0}. \tag{83}$$

This equation defines the function d_0 up to an arbitrary integration constant, because this function is a polynomial in $\ln \Lambda/p$. Therefore, due to Eq. (71) the contribution of the two-point Green function of the gauge superfield to the effective action without diagrams, containing insertions of counterterms on lines of the matter superfields, is

$$\int \frac{d^4 p}{(2\pi)^4} V(-p) \partial^2 \Pi_{1/2} V(p) \left[-\frac{1}{4e_0^2} - \frac{1}{16\pi^2} \left(\ln \frac{\Lambda}{p} - \ln G(\alpha_0, \Lambda/p) \right) + \text{const} \right]. \tag{84}$$

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However it is also necessary to take into account diagrams with insertions of counterterms. Because with the Pauli-Villars fields there are no divergences in the theory at a finite Λ, it is possible to perform the rescaling $\phi \to Z^{-1/2}\phi$, $\tilde{\phi} \to Z^{-1/2}\tilde{\phi}$. However this rescaling should be complemented by the simultaneous rescaling of Pauli-Villars fields $\Phi \to Z^{-1/2}\Phi$, $\tilde{\Phi} \to Z^{-1/2}\tilde{\Phi}$ in order to avoid appearance of the divergences in diagrams with insertions of counterterms. (Or, equivalently, in order to avoid anomalous contribution of the functional integral measure.) It is easy to see, that such rescaling is actually reduced to the substitution $m \to m/Z$, $M_i \to M_i/Z$. Making this substitution in Eq. (81) and taking into account that at $Z = 1$ the result is given by Eq. (84), at $Z \neq 1$ we obtain

$$\Gamma_V^{(2)} = \int \frac{d^4p}{(2\pi)^4}\, V(-p)\, \partial^2 \Pi_{1/2} V(p) \left[-\frac{1}{4e_0^2} - \frac{1}{16\pi^2} \ln \frac{\Lambda}{p} + \frac{1}{16\pi^2} \ln(ZG) + \text{const} \right]. \quad (85)$$

Because ZG is finite according to the definition of the renormalization constant Z, then, in order to make this expression finite, it is necessary to compensate only the one-loop divergence. For this purpose the bare charge is presented as

$$\frac{1}{e^2} Z_3\left(e, \Lambda/\mu\right) = \frac{1}{e_0^2} = \frac{1}{e^2} - \frac{1}{4\pi^2} \ln \frac{\Lambda}{\mu} + \text{const.} \quad (86)$$

Therefore, the final expression for the effective action (without gauge fixing terms) in the massless case can be written as

$$\Gamma_V^{(2)} = \int \frac{d^4p}{(2\pi)^4}\, W_a(-p)\, C^{ab} W_b(p) \left[\frac{1}{4e^2} + \frac{1}{16\pi^2} \ln \frac{\mu}{p} - \frac{1}{16\pi^2} \ln(ZG) + \text{const} \right]. \quad (87)$$

Differentiating Eq. (86) with respect to $\ln \mu$ at fixed value of the bare coupling constant e_0, we find, that the β-function, defined by

$$\beta = \frac{d}{d \ln \mu} \left(\frac{e^2}{4\pi} \right), \quad (88)$$

is

$$\beta(\alpha) = \frac{\alpha^2}{\pi}. \quad (89)$$

Such result completely agrees with the structure of the anomalies supermultiplet, which implies, that this β-function is exhausted at the one-loop.

Moreover it is possible to define Gell-Mann-Low β-function as follows: If

$$\Gamma_V^{(2)} = \frac{1}{16\pi} \int \frac{d^4p}{(2\pi)^4}\, W_a(-p)\, C^{ab} W_b(p)\, d^{-1}(\alpha, \mu/p). \quad (90)$$

(unlike Eq. (71) this expression includes contributions of diagrams with insertions of counterterms), then we define

$$\tilde{\beta}\left(d(\alpha, x)\right) \equiv -\frac{\partial}{\partial \ln x}\, d(\alpha, x) \Big|_{x=1}. \quad (91)$$

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According to Eq. (87)

$$d^{-1}(\alpha, \mu/p) = \frac{1}{\alpha} + \frac{1}{\pi} \ln \frac{\mu}{p} - \frac{1}{\pi} \ln(ZG) + \text{const.} \tag{92}$$

Differentiating this expression with respect to $\ln \mu$, we obtain

$$\tilde{\beta}(\alpha) = \frac{\alpha^2}{\pi} \Big(1 - \tilde{\gamma}(\alpha)\Big), \quad \text{where} \quad \tilde{\gamma} \equiv \frac{\partial}{\partial \ln x} \ln(ZG), \tag{93}$$

where we take into account, that the product ZG depends on α and $x \equiv \mu/p$. We see, that, unlike β, $\tilde{\beta}$ has corrections in orders of the perturbation theory.

2. Massive case:

In the massive case we will take into account diagrams with insertions of counterterms on lines of the matter superfields from the beginning. Therefore

$$I = -\frac{1}{32\pi^2} \frac{d}{d\ln\Lambda} \Bigg\{ \Bigg(\ln\Big(q^2 G^2 + m^2 J^2/Z^2\Big) + X(q^2, m/Z) \Bigg) -$$
$$- \sum_i c_i \Bigg(\ln\Big(q^2 G_{PV}^2 + M_i^2 J_{PV}^2/Z^2\Big) + X(q^2, M_i/Z) \Bigg) \Bigg\} \Bigg|_{q=0} = \frac{1}{16\pi^2}, \tag{94}$$

where e.f. $G = G(q, m/Z, M_i/Z)$, if $G(q, m, M_i)$ is a result of calculation of diagrams without insertions of counterterms on lines of the matter superfields e.t.c. Then from Eq. (76) we find

$$\Gamma_V^{(2)} = \int \frac{d^4p}{(2\pi)^4} V(-p) \partial^2 \Pi_{1/2} V(p) \left[-\frac{1}{4e_0^2} - \frac{1}{16\pi^2} \ln \frac{\Lambda}{p} + \text{finite terms} \right]. \tag{95}$$

Because, as earlier, this expression diverges (at fixed e_0) only in the one-loop approximation, in the massive case the bare charge is also presented in form (86) and β-function (88) is given by Eq. (89). As earlier this result completely agree with the structure of the anomalies supermultiplet and with the Adler-Bardeen theorem.

Note, that in the massive case it is not necessary to discuss Gell-Man-Low function, because defining this function the masses are considered to be negligibly small and the original massive theory is actually changed by the massless one.

7 Conclusion

In this paper some contributions to the two-point Green function of the gauge field at $p \to 0$ were calculated using Schwinger-Dyson equations and Ward identities for the $N = 1$ supersymmetric electrodynamics. Unfortunately we did not manage to find this function completely from Ward identities, but the most involved diagrams had been calculated completely.

Nevertheless, using the results of explicit three-loop calculations it is possible to suggest structure of terms, which can not be found from the Ward identity. Then

the β-function and Gell-Mann-Low function can be found exactly to all orders of the perturbation theory. The obtained results allow to give a simple and clear solution of the anomaly puzzle in the $N = 1$ supersymmetric electrodynamics, which, in particular, relates different solutions of the anomaly puzzle known in the literature [22]. Let us remind its main points:

Because the axial anomaly and the anomaly of the energy-momentum tensor trace are components of a single supermultiplet, β-function (88) should be exhausted at the one-loop according to Adler-Bardeen theorem [1] and is given by Eq. (89). This is the result, that is found with the higher derivative regularization both for the massless and for the massive theory. Therefore, the obtained results completely agree with the structure of the anomalies supermultiplet.

Nevertheless, Gell-Mann-Low function, which is defined by the transversal part of the two-point Green function of the gauge field, has corrections in all orders and coincides with the exact NSVZ β-function. Note, that it is sensible to define Gell-Mann-Low function only for the massless theory, because in order to construct it, it is necessary to neglect the dependence of the effective action on the mass.

Two β-functions, defined by the different ways, are different because generating functional (11) depends on the normalization point μ at fixed bare coupling constant e_0 [15, 22]. The matter is that two definitions, given above, are equivalent only if such dependence is absent. Really, if the generating functional does not depend on μ, the function $d(\alpha, \Lambda/\mu)$, defined by Eq. (90), also will not depend on μ. Differentiating this function with respect to $\ln \mu$, we obtain

$$0 = \tilde{\beta}\Big(d(\alpha, x)\Big) - \beta(\alpha)\frac{\partial}{\partial \alpha}d(\alpha, x). \tag{96}$$

In particular, at $x = 1$

$$\tilde{\beta}(\tilde{\alpha}) = \beta(\alpha)\frac{d\tilde{\alpha}}{d\alpha}, \tag{97}$$

where $\tilde{\alpha} \equiv d(\alpha, 1)$. It means, that two different definitions of the β-function are equivalent. However in the considered case generating functional (11) depends on the normalization point μ at the fixed e_0. Really, the dependence on μ in Eq. (11) comes from the μ-dependent renormalization constant Z. One can try to remove this dependence by the substitution $\phi \rightarrow Z^{-1/2}\phi$. However, according to [14] this substitution has an anomalous Jacobian, which contains $\ln Z$, depending on μ. (This Jacobian was explicitly written in [15, 22].) Therefore, even after the substitution $\phi \rightarrow Z^{-1/2}\phi$ generating functional (11) will contain $\ln Z$ and, therefore, will depend on μ. As a consequence Eq. (97), which relates β-functions defined by the different ways now is not valid, and the functions β and $\tilde{\beta}$ are not equivalent in the considered case.

In principle it is possible to define the generating functional, which does not depend on the normalization point. Two different ways to do it were proposed in [22]. Then β-function (88) has corrections in all orders of the perturbation theory, but the arguments, based on the structure of the anomalies supermultiplet fail due to some reasons. That

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[1]With the higher derivative regularization this theorem is valid, while with the dimensional reduction it seems to fail [25].

is why the anomaly puzzle does not take place if the theory is regularized by higher derivatives.

Finally we mention, that it would be interesting to perform the similar investigation for the supersymmetric Yang-Mills theory. However for this theory the calculations with the higher covariant derivative regularization are very involved, because the presence of the term with higher *covariant* derivatives [26] essentially complicates the form of the action. In this case the calculations can be considerably simplified if the covariant derivatives are substituted by the ordinary derivatives. Then the gauge invariance will be certainly broken, so that Ward identities can be also broken by some local noninvariant terms. Nevertheless the gauge invariance can be restored by the special choice of the subtraction scheme, so that it is possible to use noninvariant regularizations [27, 28] for the calculations. Such scheme was proposed in [29] for Abelian supersymmetric theories and in [30] for the nonabelian theories. So, it is possible, that application of the above described regularization to supersymmetric Yang-Mills theories enables us to perform similar calculations also in the nonabelian case. At present this work is in progress.

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References

[1] *S.Ferrara, B.Zumino*, Nucl.Phys. **B 87**, (1975), 207.

[2] *T.E.Clark, O.Piquet, K.Sibold*, Nucl.Phys. **B 143**, 445, (1978).

[3] *O.Piquet, K.Sibold*, Nucl.Phys. **B 196**, 428, (1982).

[4] *O.Piquet, K.Sibold*, Nucl.Phys. **B 196**, 447, (1982).

[5] *S.L.Adler, W.A.Bardeen*, Phys.Rev. **182**, 1517, (1969).

[6] *L.D.Faddeev, A.A.Slavnov*, Gauge fields, introduction to quantum theory, second edition, Benjamin, Reading, 1990.

[7] *S.L.Adler, J.C.Collins, A.Duncan*, Phys.Rev. **D 15**, 1712, (1977).

[8] *V.A.Novikov, M.A.Shifman, A.I.Vainstein, V.I.Zakharov*, Phys.Lett. **157B**, (1985), 169.

[9] *W.Siegel*, Phys.Lett. **84 B**, (1979), 193.

[10] *O.V.Tarasov, V.A.Vladimirov*, Phys.Lett. **96 B**, 94, (1980).

[11] *M.T.Grisaru, M.Rocek, W.Siegel*, Phys.Rev.Lett. **45**, 1063, (1980).

[12] *W.Caswell, D.Zanon*, Phys.Lett. **100 B**, 152, (1980).

[13] *M.Shifman, A.Vainstein*, Nucl.Phys. **B277**, (1986), 456.

[14] *N.Arkani-Hamed, H.Mirayama*, JHEP **0006**, (2000), 030, (hep-th/9707133).

[15] *K.Stepanyantz*, Theor.Math.Phys. **140**, (2004), 939 (hep-th/0301167).

[16] *W.Siegel*, Phys.Lett. **94B**, (1980), 37.

[17] *G.t'Hooft, M.Veltman*, Nucl.Phys. **B44**, (1972), 189.

[18] *A.A.Slavnov*, Theor.Math.Phys. **23**, (1975), 3.

[19] *T.Bakeyev, A.Slavnov*, Mod.Phys.Lett. **A11**, (1996), 1539.

[20] *A.Soloshenko, K.Stepanyantz*, Two-loop renormalization of $N = 1$ supersymmetric electrodynamics, regularized by higher derivatives, hep-th/0203118.

[21] *A.Soloshenko, K.Stepanyantz*, Theor.Math.Phys., **134**, (2003), 377.

[22] *A.Soloshenko, K.Stepanyantz*, Teor.Mat.Fiz. **140**, (2004), 437 (hep-th/0304083).

[23] *V.Novikov, M.Shifman, A.Vainstein, V.Zakharov*, Phys.Lett. **166B**, (1985), 329.

[24] *P.West*, Introduction to supersymmetry and supergravity, World Scientific, 1986.

[25] *D.I.Kazakov*, JETP Lett. **41**, (1985), 335.

[26] *P.West*, Nucl.Phys. **B 268**, (1986), 113.

[27] *A.A.Slavnov*, Phys.Lett. **B 518**, (2001), 195.

[28] *A.A.Slavnov*, Theor.Math.Phys. **130**, (2002), 1.

[29] *A.A.Slavnov, K.V.Stepanyantz*, Theor.Math.Phys., **135**, (2003), 673.

[30] *A.A.Slavnov, K.V.Stepanyantz*, Theor.Math.Phys., **139**, (2004), 599.
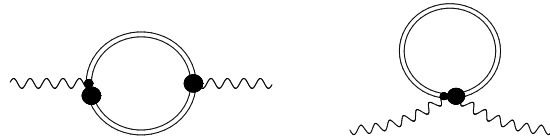
Figure 1: Feynman diagrams, defining the β-function of $N = 1$ supersymmetric electrodynamics.